EDEXCEL 1387
Summer 2004
HIGHER SOLUTIONS
Paper 5 (Non-calculator)

1.

(a) (i) The decimal point has moved back in both numbers, therefore it should move back twice in 221

= 2.21

(ii) I would firstly rearrange the sum given into the style of the question $\rightarrow \frac{221}{17} = 13$

Compare what you have with $\frac{221}{17} = 13$, its $\frac{22.1}{1700} = ?$

Back one decimal place on the top and forward two on the bottom, but because its divide it also means back two $\rightarrow$ Altogether back 3

= 0.013

(b) You have the prime factors, $39 = 13 \times 3$ and $17 = 17$, no factors are present in either so, $13 \times 3 \times 17 = 663$

2.

$\frac{\pi abc}{2d}$	πa^3	$2a^2$	$\pi a^2 + b$	$\pi(a+b)$	$2(c^2+d^2)$	$2ad^2$
✓	✗	✓	✗	✗	✓	✗

Ignore π and 2, then cancel '*d*' with any letter from the top. Leaves you with length $\times$ width which in effect is an area.

$(\text{Area} + \text{Area}) = \textit{bigger}$ area

3.

(a) About 200×0.2, which can also be written as $200 \times \frac{1}{5} \rightarrow$ Multiply across

$\rightarrow \frac{200}{5} = 40$

(b) Remember OR means addition when referring to probability $\rightarrow 0.2 + 0.4$

= 0.6

4.

(a) Just divide 108 by the lowest prime number you can think of each time

$108 \div 2 = 54$

$54 \div 2 = 27$

$27 \div 3 = 9$

$9 \div 3 = 3$

$3 \div 3 = 1$

Hence, 108 is the same as $2^2 \times 3^3$

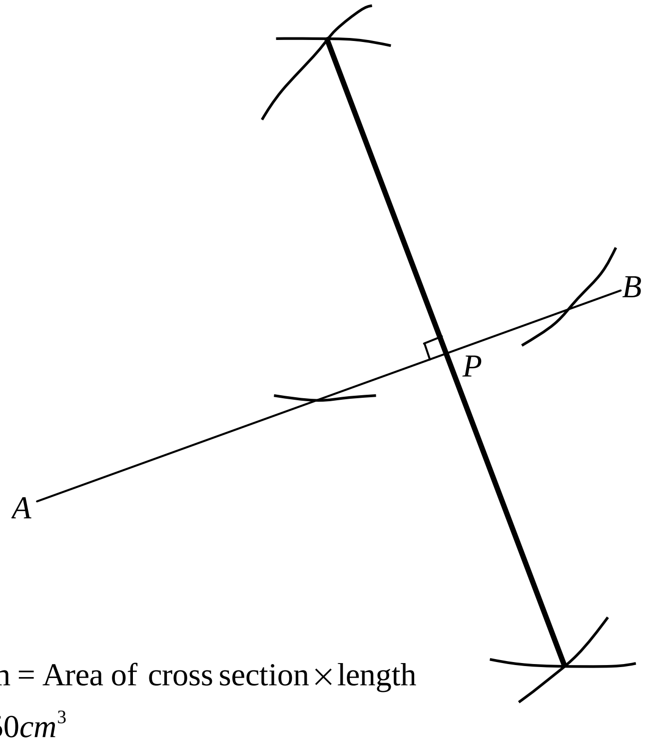
(b) Using the same method in part (a), 24 can be written as $2^3 \times 3$

Compare $2^2 \times 3^3$ with $2^3 \times 3$

2^2 is in both and 3, hence HCF = $2^2 \times 3$

$= 12$

5. See diagram:



6. Volume of a prism = Area of cross section × length

$15cm^2 \times 10cm = 150cm^3$

7.

(a) $= k^{5-2}$

$\rightarrow k^3$

(b) *(i)* $4x+20+3x-21$

$\rightarrow 7x-1$

(ii) Using FOIL (First Outer Inner Last)

$\rightarrow x^2+3xy+2xy+6y^2$

$\rightarrow x^2+5xy+6y^2$

(c) $(p+q)(p+q+5)$

(d) Bracket to a power...just multiply the powers

$m^{-4\times-2} \rightarrow m^8$

(e) $6t^{2+4}r^3 \rightarrow 6t^6r^3$

8. If it is to the nearest mm, then it is either ± 0.5mm.

(i) $101mm - 0.5mm$

$= 100.5mm$

(ii) $101mm + 0.5mm$

$= 101.5mm$

9. Firstly it would be a good idea to work out the area of the triangle.

Area of a triangle $= \frac{1}{2}(base \times height)$

$\rightarrow \frac{1}{2}\left(\frac{5}{8}\times 6\frac{2}{5}\right)$ I think here it would be a good idea to rewrite $6\frac{2}{5}$ as a top heavy fraction so it becomes clearer to work out

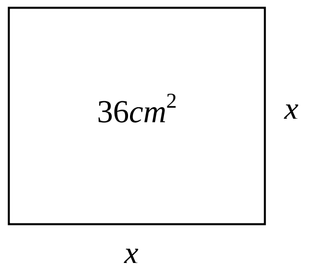
$\rightarrow \frac{1}{2}\left(\frac{5}{8}\times\frac{32}{5}\right)$

$\rightarrow \frac{1}{2}\left(\frac{160}{40}\right) \rightarrow \frac{1}{2}(4) \rightarrow 2cm^2$

From information in the question, area of the square is 18 times bigger, therefore $\rightarrow$

$2cm^2 \times 18 = 36cm^2$

Think about the square now. If one side of the square is *x*, then all the other sides must also be *x*.



The area of a square is length $\times$ width. As shown in the diagram, this is equal to x^2, hence $x^2 = 36$, therefore $x = \pm 6$. Only the positive value counts here as x is a length.
The perimeter is 4 times $x \rightarrow 24cm$

10.

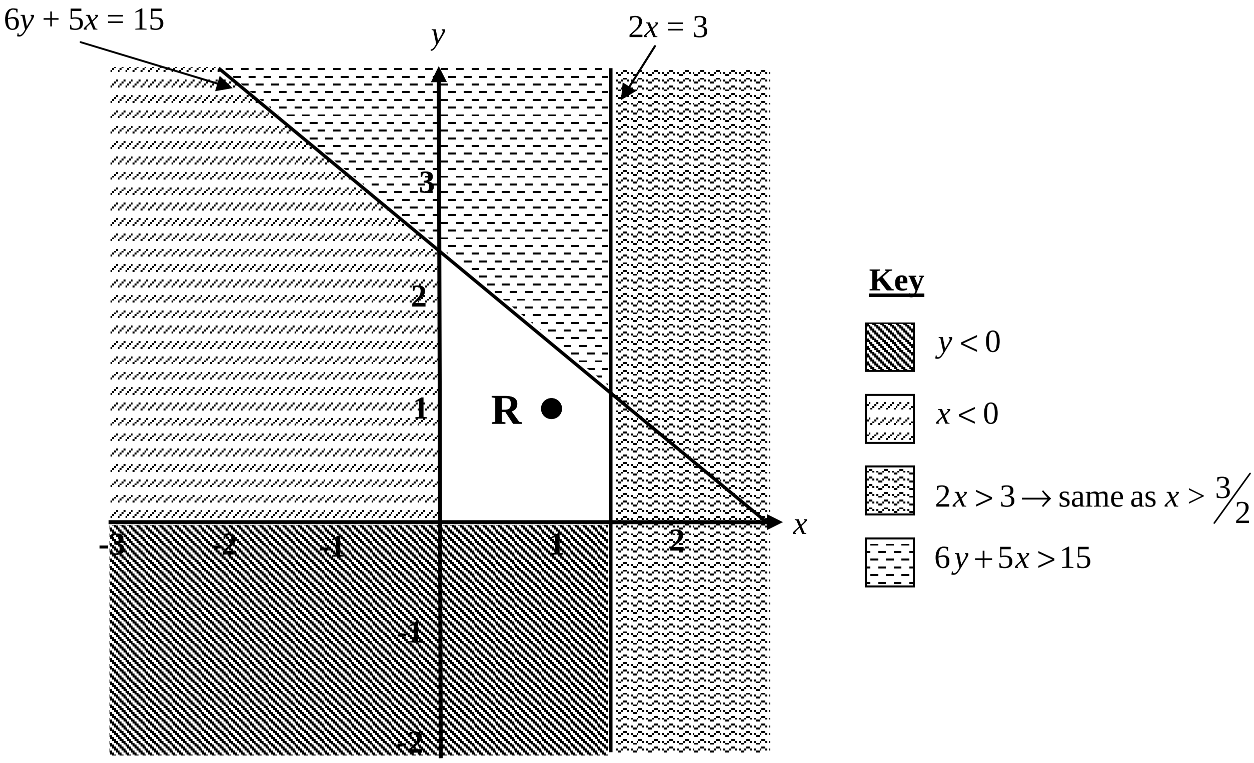
(a) $6y+5x=15 \rightarrow 6y=15-5x \rightarrow y = \frac{1}{6}(15-5x)$ OR $y = \frac{15-5x}{6}$

(b) Subs $x = -21$ into equation $\rightarrow y = \frac{1}{6}(15-5(-21))$

$\rightarrow y = \frac{1}{6}(120) \rightarrow y = 20$

hence, k = 20

(c) (i) See diagram



(ii) P (1,1)

11.

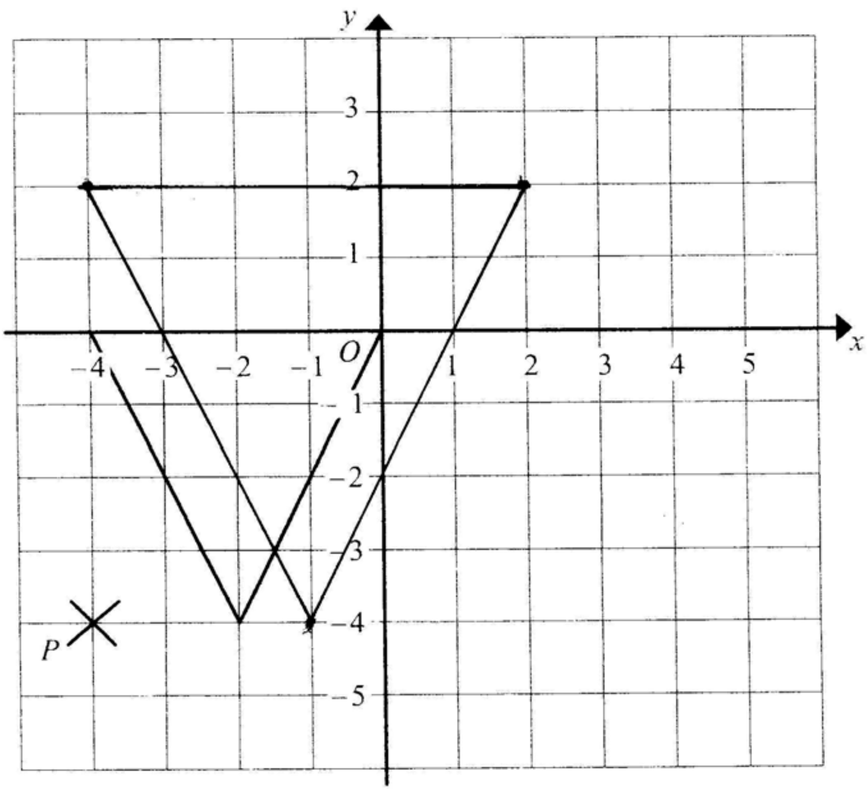
(a) The line DC is parallel to AB, hence they have the same gradient.
As you can see on the diagram, the line DC has a y intercept of 6 (at C),
therefore the equation of the line will be:
$\rightarrow y = 2x + 6$

(b) The line BC is perpendicular to AB. This means the line BC intersects with AB at 90°. A simple formula to work out the gradient of the perpendicular is, $m_1 m_2 = -1$, where m_1 = the gradient you know (in this case of AB) and m_2 is the gradient of the perpendicular. So if we plug the numbers into the formula....$(2)m_2 = -1 \rightarrow m_2 = -\frac{1}{2}$
We know the line has a y intercept of 6 (at C) and we now know the gradient, so the equation of the line is:
$\rightarrow y = -\frac{1}{2}x + 6$

(c) The diagonal of the rectangle, in this case AC, becomes the diameter of the circle and then using the circle property '*angles in a semicircle*' this means that the circle will go through the other two vertices

12.

See diagram

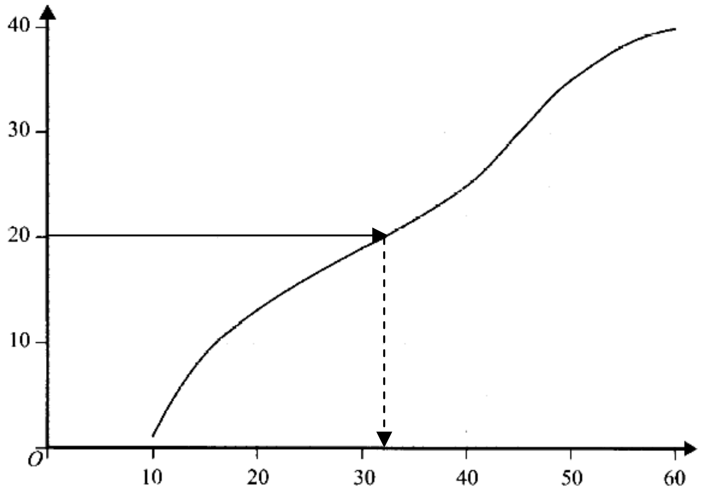


Enlarge the shaded traiangle by scale factor 1½, centre P.

13.

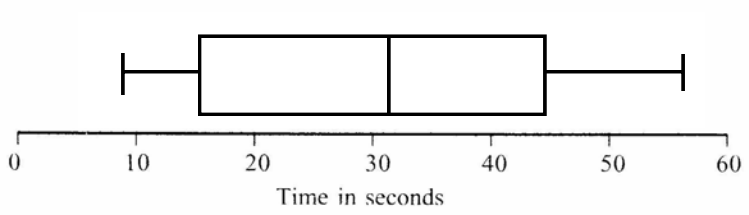
(a) Median time will occur at when the CF $\rightarrow \frac{40}{2} = 20$

Draw a line across as shown in the diagram and read off the time



$\rightarrow$ *about 32 seconds*

(b) See box plot



[Note that your median on the diagram has to be consistent with part (a)]

(c)

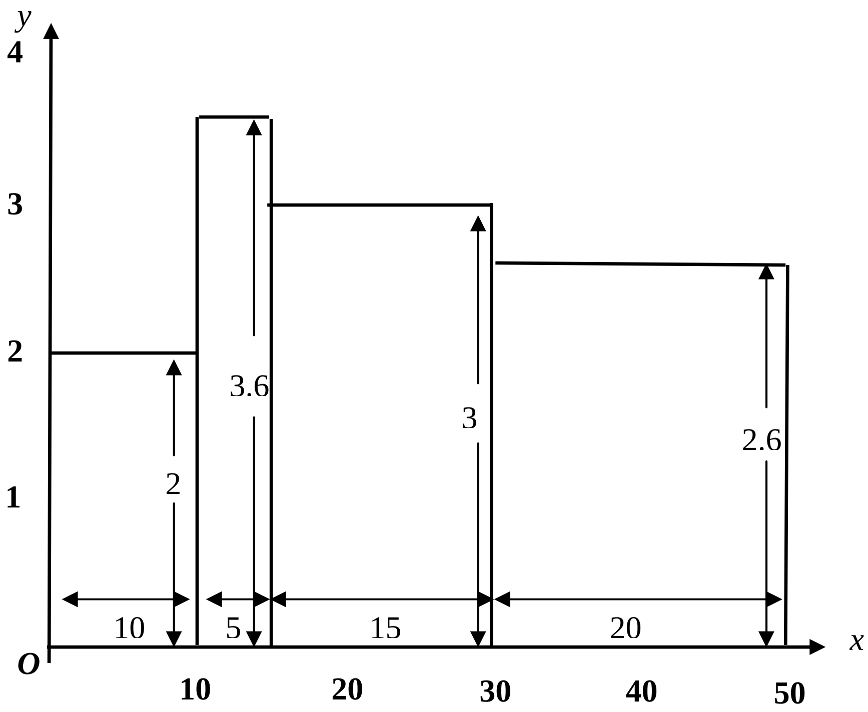
- The boys data is more spread out than the girls, it has a higher IQR than the girls
- The girls median is lower than the boys, 30 seconds rather than 32 seconds
- The girls data has a smaller range than that of the boys

14.

Remember with histograms, it's the *area* of the bar that counts

See diagram below for further understanding of what is meant here

Time (t minutes)	Frequency
$0 < t \leq 10$	$2 \times 10 = 20$
$10 < t \leq 15$	$3.6 \times 5 = 18$
$15 < t \leq 30$	$3 \times 15 = 45$
$30 < t \leq 50$	$2.6 \times 20 = 52$
	$\Sigma = 135$



15. V = pq^t

(a) $1600 = pq^0 \rightarrow$ Anything to the power $0 = 1$ so... $\rightarrow 1600 = p$

$$400 = 1600q^2 \rightarrow \frac{400}{1600} = q^2 \rightarrow \frac{4}{16} = q^2, \text{ hence } q = \sqrt{\frac{4}{16}} \rightarrow q = \frac{\sqrt{4}}{\sqrt{16}} \rightarrow \frac{2}{4} \rightarrow \frac{1}{2}$$

$p = 1600$ and $q = \frac{1}{2}$

(b) V = pq^t

$$\text{V} = (1600)\left(\frac{1}{2}\right)^{-2}$$

$$\rightarrow 1600 \times \frac{1}{\left(\frac{1}{2}\right)^2}$$

$$\rightarrow 1600 \times \frac{1}{\frac{1}{4}}$$

$$\rightarrow 1600 \times 4 = £6400$$

16.

(a) In this question I feel its best to use symmetry.

PB = PC $\rightarrow$ Tangents to a circle are equal.

AB = AC as the triangle is isosceles and PA is common giving

Side Side Side congruency (SSS)

Why is it not SAS?

If you had not got the fact that ABC = ACB then A could be anywhere on the circumference. Therefore there is *no* reason at all that the angles in the lower triangles are right angles, even if it they 'appear' it. You are *not* told that AP bisects BPC.

In ABP and ACP, you can deduce AB = AC (from isosceles triangle as ABC = ACB). AP is common. BP = CP (tangent property) so ABP is congruent to ACP hence giving, SSS congruency.

Call the point where BC cuts AP, O

In either BAO, CAO or BPO and CPO you can deduce angle BAO = CAO or BPO = CPO and hence triangle BAO is congruent to CAO (SAS) and hence AOB = AOC, but AOB + AOC = 180 therefore AOB = AOC = 90° or similar from BPO and CPO.

You can't prove BOP = 90 by looking at it or assuming symmetry until at least you have proved congruence of BAP and CAP. Otherwise you are basically assuming what you are trying to prove!

(b) Call the point where BC cuts AP, O

$P\hat{O}B = 90°$

$P\hat{B}O = 90 - 10 \rightarrow 80°$

Using Alternate segment theorem, $B\hat{A}C = 80°$

$\rightarrow B\hat{A}O = 40°$

hence, $A\hat{B}C = 90 - 40 \rightarrow 50°$

17.

(a) There are a couple of ways to answering this question, here is one way.

$\overrightarrow{OP} = \overrightarrow{OA} + \frac{2}{3}\overrightarrow{AC}$, where $\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC}$

$\rightarrow 6a + \frac{2}{3}(-6a + 6c)$

$\rightarrow 6a + (-4a + 4c)$

$\rightarrow 2a + 4c$

$\rightarrow \overrightarrow{OP} = 2(a + 2c)$

(b) If $\overrightarrow{OP} = 2(a + 2c)$, then for $\overrightarrow{OPM}$ to be a straight line, $\overrightarrow{OM}$ should be some constant$\times(a + 2c)$

Show this $\rightarrow \overrightarrow{OM} = \overrightarrow{OP} + \overrightarrow{PC} + \overrightarrow{CM}$

$\rightarrow 2(a + 2c) + \left(\frac{1}{3}(-6a + 6c)\right) + 3a$

$\rightarrow 2a + 4c - 2a + 2c + 3a$

$\rightarrow 3a + 6c \rightarrow 3(a + 2c)$ as required

18.

(a) $16^{\frac{1}{2}}$ is the same as saying $\sqrt[2]{16} \rightarrow \pm 4$

(b) $\sqrt{40}$ can also be written as $\sqrt{4 \times 10}$, when the 4 is 'taken out', it is square rooted, therefore $\rightarrow 2\sqrt{10}$,

$k = \pm 2$

(c) Area of the large rectangle is equal to $\left(\sqrt{5} + \sqrt{20}\right)\left(\sqrt{8}\right) \rightarrow \sqrt{40} + \sqrt{160}$

$\rightarrow$ Using the same method in part (b), $\sqrt{40} = 2\sqrt{10}$ and $\sqrt{160} = 4\sqrt{10}$

$\rightarrow 2\sqrt{10} + 4\sqrt{10} = 6\sqrt{10}$

Area of the little rectangle = $\sqrt{5} \times \sqrt{2} = \sqrt{10}$

Area of big rectangle once little rectangle is removed = $6\sqrt{10} - 1\sqrt{10} \rightarrow 5\sqrt{10}$

....as a '%' of the big rectangle $\rightarrow \dfrac{5\sqrt{10}}{6\sqrt{10}} \times 100\%$ ($\sqrt{10}$ cancels with the other)

$\rightarrow \dfrac{5}{6} \times 100 \rightarrow \dfrac{500}{6} \rightarrow 83\frac{1}{3}\%$

19.

(a) (i) There are different ways to factorise this quadratic, the method I use is known as 'splitting the middle term'.

Which two values multiply to make $196\ (2\times 98)$ and add to make -35?

$\rightarrow -7$ and -28

Now look at the eqation in two parts....

$\rightarrow\ 2x^2-28x \qquad -7x+98$

Factorise each separately

$\rightarrow 2x(x-14) \qquad -7(x-14)$

hence there is a common bracket. The other bracket consists of the values outside the common bracket, like this $\rightarrow (2x-7)(x-14)$

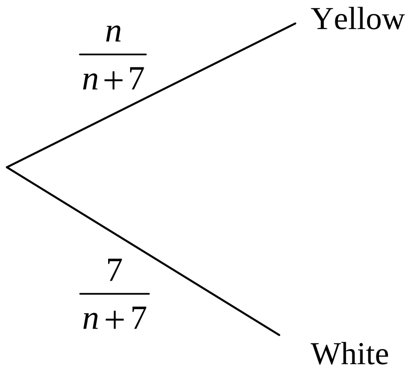
(ii) $(2x-7)(x-14)=0$

$(2x-7)=0 \rightarrow x=3.5$

OR $(x-14)=0 \rightarrow x=14$

(b)(i) See tree diagram

$=\dfrac{7}{n+7}$



(ii) This implies $\dfrac{7}{n+7}=\dfrac{2}{5}$

$\rightarrow 35=2(n+7)$

$\rightarrow 35=2n+14$

$\rightarrow 21=2n$

$\rightarrow n=10.5$, n has to be an integer therefore Bill's statement is wrong *(c)* Look at the tree diagram to help you if you don't follow this bit.

$$\rightarrow\left(\frac{7}{n+7}\times\frac{n}{n+7}\right)+\left(\frac{n}{n+7}\times\frac{7}{n+7}\right)=\frac{4}{9}$$

$$\rightarrow\frac{7n}{(n+7)^2}+\frac{7n}{(n+7)^2}=\frac{4}{9}$$

$$\rightarrow\frac{14n}{(n+7)^2}=\frac{4}{9}$$

$$\rightarrow 126n=4(n+7)^2$$

$$\rightarrow 126n = 4(n^2 + 14n + 49)$$

$$\rightarrow 126n = 4n^2 + 56n + 196$$

$$\rightarrow 4n^2 - 70n + 196 = 0$$

Hence, $\tfrac{1}{2}(4n^2 - 70n + 196 = 0) \rightarrow 2n - 35n + 98 = 0$

(d) Using part *(a)(ii)* or otherwise, this means n must equal 14, as n HAS to be an integer

$$\left(\frac{7}{n+7}\times\frac{7}{n+7}\right)\rightarrow\frac{49}{(n+7)^2}$$

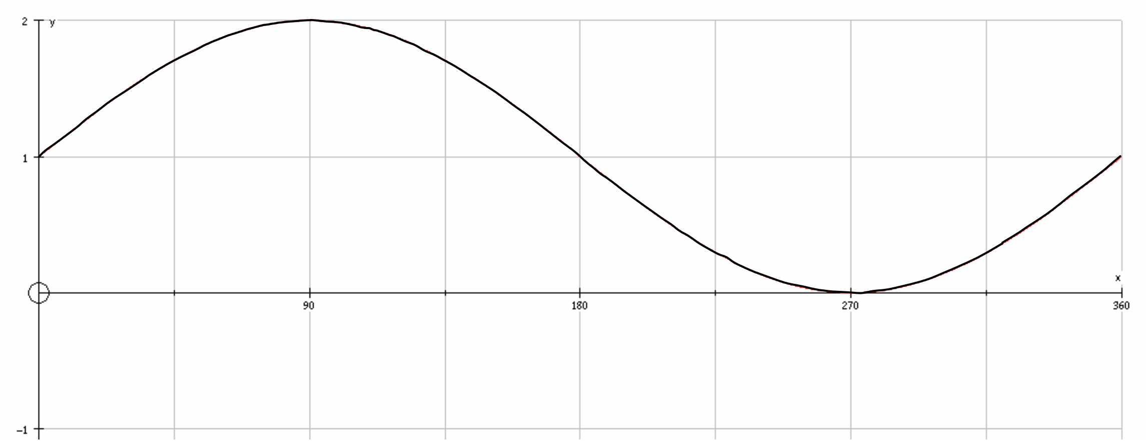
If $n=14$. then,

$$\rightarrow\frac{49}{(14+7)^2}\rightarrow\frac{49}{21^2}\rightarrow\frac{49}{441}\rightarrow\frac{1}{9}$$

20.

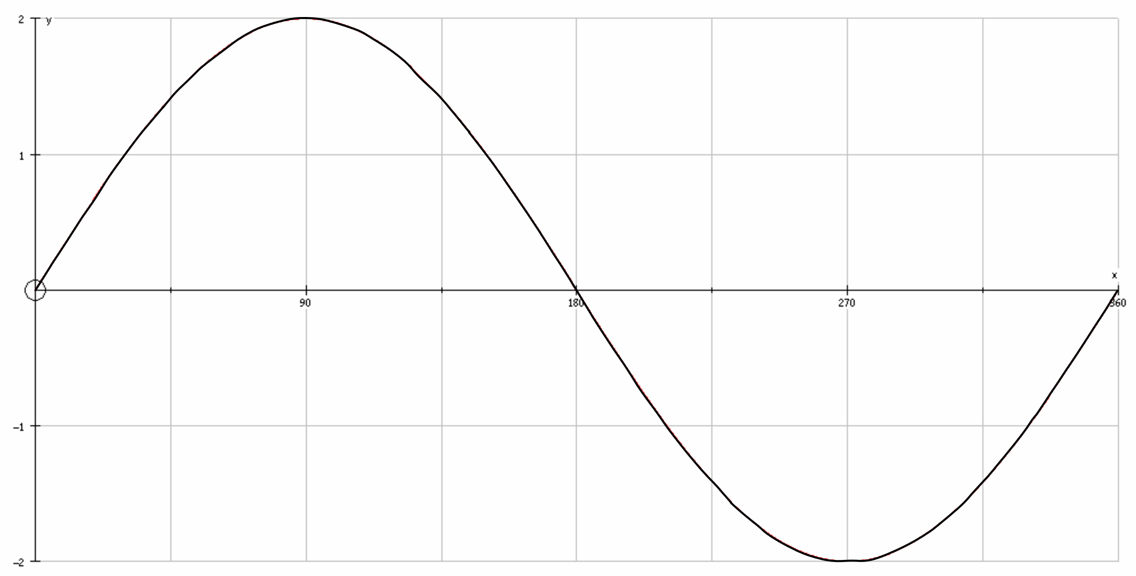
(a)(i) Entire curve has shifted up the y axis by 1 unit

$$\rightarrow y = \sin x + 1$$



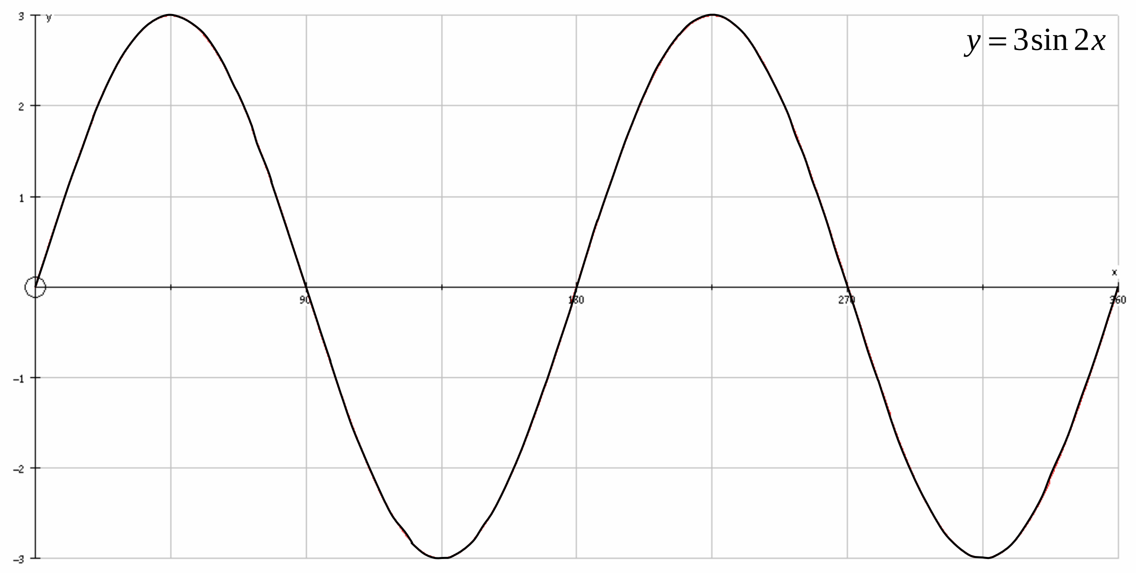
(ii) The curve is stretched by scale factor 2 parallel to the y axis,

$\rightarrow y = 2\sin x$



(iii) • The curve is stretched by scale factor $\frac{1}{2}$ parallel to the x axis

• The curve is stretched by scale factor 3 parallel to the y axis



END OF PAPER 5 SOLUTIONS