Estimate Probability using Relative Frequency & Use Tree Diagrams HD2

Express probabilities as fractions, decimals or percentages ONLY

The Words

Mutually exclusive means events that cannot occur together e.g. *Head* and *Tail* when a coin lands, or to describe the outcomes at the **end of a tree diagram**. When results are **mutually exclusive** we **add** probabilities. *See how the four outcome probabilities in example 3 below are added to make 1.*

Events are **independent** when one event does not effect the outcome of the other. Independent events may or may not occur together. These are easy to spot because the events are usually separated 'physically' or by 'time'. For example, the results from two dice are two independent events **separated physically**. Catching the bus in the morning and catching the bus in the evening are two independent events **separated in time**. When results are **independent** we **multiply**. **This may seems a bit complicated but remember in examinations questions with tree diagrams it just means <u>we multiply along the branches</u>.** *See example 3 below.*

Relative frequency refers to the ratio of successes to tries in an **experiment** and is used to **estimate** probability.

The Formulae - Memorise These

$$\text{P(Event)} = \frac{\text{The Number of Possible Successes}}{\text{The Total Number of Possible Ways}} \qquad \text{Relative Frequency} = \frac{\text{The Number of Successes}}{\text{The Number of Trials}}$$

P(not A) = 1 – P(A)

P(A **or** B) = P(A) **+** P(B) - where A and B cannot occur together - mutually exclusive.

P(A **and** B) = P(A) × P(B) - for independent events.

Example 1: Adding

Calculate the probability that an even number or a 3 lands on a fair die. An even number and a 3 cannot occur together, therefore P(Even OR 3) = P(Even) + P(3) = $\frac{1}{2} + \frac{1}{6} = \frac{3}{6} + \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$.

Example 2a: Estimate Relative Frequency

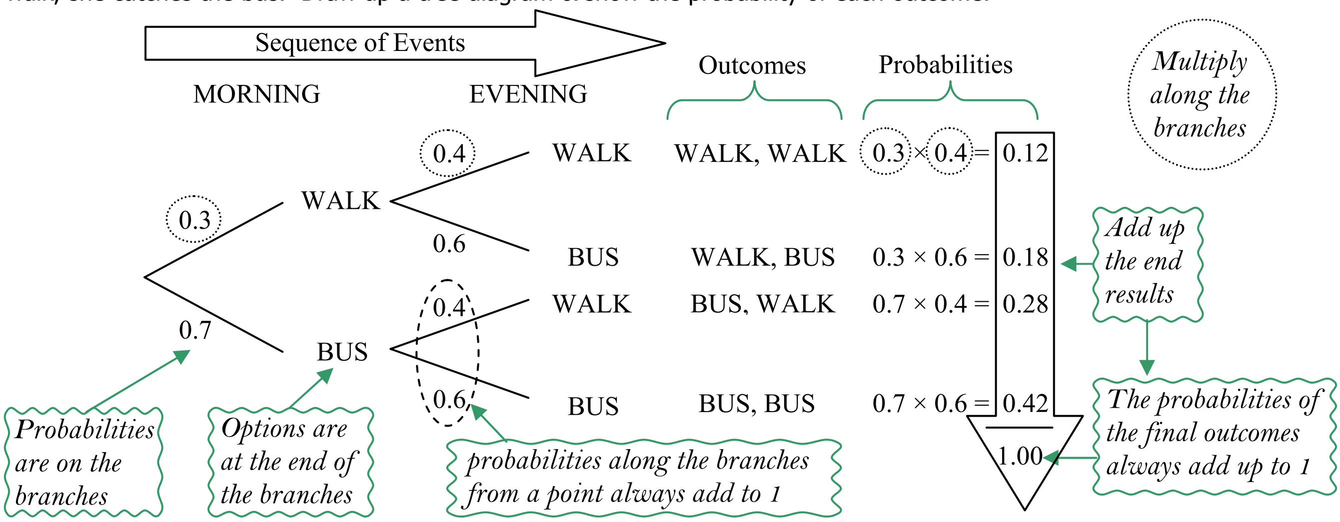
A biased die is thrown 100 times and lands on two 10 times; estimate the probability that the die will land on two on its next throw. Estimate for P(two) = $\frac{\text{The Number of Successes}}{\text{The Number of Trials}} = \frac{10}{100} = \frac{1}{10}$.

Example 2b: Estimate the number of results using Relative Frequency

The biased die is now thrown 450 times. Estimate the number of times it will land on two? $450 \times \frac{1}{10} = 45$.

Example 3: Tree Diagram

The chance Amy walks to school in the morning is 0.3; the chance she walks home in the evening is 0.4. If she does not walk, she catches the bus. Draw up a tree diagram & show the probability of each outcome.



Your Turn!!

a) John randomly selects 20 students in the school hall and finds that 3 are year 7 students. There are 500 students in the hall. Estimate how many students in the hall are year 7 students.

b) In example 3, what is the probability that Amy catches the bus both in the morning & in the evening?

c) If Amy changes her habits such that chance she walks in the morning is 0.1 and 0.2 in the evening. Copy the tree diagram above but with the correct probabilities on the branches. What are final four probabilities that add up to 1?

RAPID 'ACID' TEST part i) – Blank out the page above before answering these!

1. After travelling through a town ten times, you notice that the first set of traffic lights were green three times and the second set were green six times. Given that the traffic lights work independently, draw a tree diagram with the options 'green' and 'not green' to show the possible outcomes and the estimated probabilities of these outcomes. Include the probabilities of the final outcomes e.g. green followed by green. *HD2 continued on next page*

Multiply Probabilities for Several Independent Events Knowing when to ADD (+), TAKE (–), and MULTIPLY (×) (HD2)

The Words

Remember what we said about independent events on the previous page…

Events are **independent** when one event does not effect the outcome of the other. Independent events may or may not occur together. These are easy to spot because the events are usually separated 'physically' or by 'time'. For example, the results from two dice are two independent events **separated physically**. Catching the bus in the morning and catching the bus in the evening are two independent events **separated in time**. When results are **independent** we **multiply**.

And the formulae…

P(not A) = 1 – P(A)

P(A **or** B) = P(A) **+** P(B) - where A and B cannot occur together - mutually exclusive.
P(A **and** B) = P(A) × P(B) - for independent events.

Example 4: Independent Events
The probability that I catch the bus in the morning is 0.3 and in the evening is 0.5. Assuming that these events are independent, calculate the probability that I a) catch the bus in the morning and the evening.
b) catch the bus 3 mornings in a row - assume independence between days.

a) P(Morning AND Evening) = P(Morning) × P(Evening) = 0.3 × 0.5 = 0.15.
b) P(Catch in Morning 3 days in a row) = P(Morning) × P(Morning) × P(Morning) = 0.3 × 0.3 × 0.3 = 0.027

Common Examination Question

Example 5: Knowing when to Add, Take and Multiply.
John has 3 green (G) socks, 2 blue (B) socks, and 5 yellow (Y) socks.
He selects two socks. Calculate the probability he wears socks of different colours.

Solution

Assume he selects one sock and then another (without replacement)…
For different colours we need (G and then B), (B and then G), (G and then Y), (Y and then G), (B and then Y), (Y & then B).
This is a lot of different ways; it is more helpful to **first** ask the question 'What is the probability of the same colour'. For the same colour we have (G and then G), (B and then B), (Y and then Y). This is just 3 ways which we can quickly calculate.
P[(G then G) or (B then B) or (Y then Y)] = P(G and then G) + (B and then B) + (Y and then Y) *notice the + replacing **or***

After the first sock is selected green, there are only two green socks left.

= P(G) × P(then G) + P(B) × P(then B) + P(B) × P(then B) *notice the × replacing **and***

After the first sock is selected blue, there is only 1 blue sock left.

$$= \frac{3}{10}\times\frac{2}{9} + \frac{2}{10}\times\frac{1}{9} + \frac{5}{10}\times\frac{4}{9}$$

After the first sock is selected yellow, there are only 4 yellow sock left.

After the first sock is selected there are just 9 left.

$$= \frac{6}{90} + \frac{2}{90} + \frac{20}{90} = \frac{28}{90} = \frac{14}{45}$$ *For a reminder of adding & multiplying fractions see NA9*

P(he wears socks of DIFFERENT colours) = 1 – P(he wears socks of SAME colours) *notice the use of (–) to get back to the probability the question asked for.*

$$= 1 - \frac{14}{45} = \frac{31}{45}$$

Your Turn!!

d) Box A contains 10 ice creams. 1 vanilla, 2 toffee and 7 strawberry.
Box B contains 15 ice creams. 2 vanilla, 3 toffee and 10 strawberry.

Jane takes one ice cream from each box.
Work out the probability the two ice creams are not of the same type.

RAPID 'ACID' TEST part ii) – Blank out the page above before answering these!

2. The probability a spinner lands on 6 is $\frac{1}{10}$. Calculate the probability that in 5 throws, it lands on 6 five times.
3. A bag contains 2 black beads, 3 green and 5 yellow.
Jackie takes a bead, records the colour and replaces it. She does this two more times.
Calculate the probability that a) the beads are all of the same colour
b) exactly two are the same colour

Hint for b): Require P[(B,B,>B or B,>B,B or >B,B,B) or (G,G,>G or G,>G,G or >G,G,G) or (Y,Y,>Y or Y,>Y,Y or >Y,Y,Y)]
Note: in this hint >G is short hand for **not** G. Also , is short for **<u>and</u>**.
Note also: It will be quicker if you realise that P(B and B and notB) = P(B and notB and B) = P(notB and B and B).