

# Answers – Your Turn!! – Number & Algebra

## NA1 HCF LCM

- a) 24  
b) 4

## NA2 Primes

- a) Primes 2 3 5 7 11 13 17 19 23  
b)  $2 \times 2 \times 3 \times 3 \times 5 \times 7$

## NA3 Negatives

- a) 24  
b) -108  
c) -5  
d) 2  
e) 11

## NA4 Substitution

- a) Ex1)

$$y = 3 \times 2 + 7 \\ = 6 + 7 = 13$$

Ex2)

$$y = 3(2)^2 - 7(2) + 2 \\ = 3 \times 4 - 14 + 2 \\ = 12 - 12 = 0$$

- b) Ex1)

$$y = 3(-3) + 7 \\ = -9 + 7 \\ = -2$$

Ex2)

$$y = 3(-3)^2 - 7(-3) + 2 \\ = 3 \times 9 + 21 + 2 \\ = 27 + 23 \\ = 50$$

## NA5 Index Laws

- a) -  
b) i)  $3^{6-2} = 3^4$   
ii)  $4^{4 \times 6} = 4^{24}$

- c) -  
d) i)  $e^{5-2} = e^3$   
ii)  $m^{8 \times 4} = m^{32}$

## NA6 Significant Figures

- a) 3.1 2sf  
b) 3.14 3sf  
c) 3.141 4sf  
d) 0.34 2sf  
e) 0.003406 4sf

## NA7 SIF

- a) 0.0000245  
b) 31400  
c)  $1.2 \times 10^2$  (also correct  $1.20 \times 10^2$ )  
d)  $1.05 \times 10^{-2}$   
e) & f) self check

## NA8 Ratio

- a)  $78 \div 6 = 13$ ,  $3 \times 13 = 39$ ,  $2 \times 13 = 26$ ,  $1 \times 13 = 13$   
b) 2:43  
c)  $\frac{3}{8}$

## NA9 Fractions

- a)  $\frac{1}{4}$  b)  $\frac{1}{4}$   
c)  $\frac{19}{40}$   
d)  $\frac{5}{7}$   
e)  $\frac{5}{8} \times \frac{9}{5} = \frac{1}{8} \times \frac{9}{1} = \frac{9}{8}$  or  $1\frac{1}{8}$

## NA10 Fraction Neg. Powers

- a) 1 over (or under 1)  
b) root  
c)  $\frac{1}{9}$   
d)  $\frac{1}{3}$   
e) 4  
f) 3  
g) 9  
h)  $\frac{1}{9}$

## NA11 Recurring Decimals

- a)  $\frac{1234}{9999}$   
b)  $0.0\dot{3}4$   
c)  $12.99\dot{3}4\dot{5}$

- d)  $\frac{4}{9}$  e)  $\frac{1}{8}, \frac{1}{25}$

## NA12 %'s

- a) i)  $0.15 \times 12 = \text{£}1.80$   
ii)  $\text{£}1.80 \times 5 = \text{£}9$   
b)  $0.15 \times 13 = 1.95$   
 $13 - 1.95 = \text{£}11.05$   
c) i)  $200 \times (1.1)^2 = \text{£}242$   
ii)  $200 \times (1.1)^{50} = \text{£}23478.17$  nearest penny

## NA13 Reverse %'s

- a)  $\text{£}6166$  to nearest  $\text{£}$   
b)  $\text{£}341.18$  to nearest penny

## NA14i) Direct Proportion

- a) 10  
b) i) 4  
ii) 20  
c)  $y = kx$ ,  $80 = k5$ ,  $k = 16$ ,  $y = 16x$   
 $x = 2$ ,  $y = 16 \times 2 = 32$   
d) 10  
e)  $y = kx$ ,  $7 = k24$ ,  $k = \frac{7}{24}$ ,  $y = \frac{7}{24}x$   
 $y = 8$   $8 = \frac{7}{24}x$ ,  $x = \frac{8 \times 24}{7} = 27\frac{3}{7}$   
(or  $x = 27.428\dots$ )

## NA14ii) Inverse Proportion

- a) 200  
b) 100  
c) i)  $y = \frac{k}{x}$ ,  $11 = \frac{k}{2}$ ,  $k = 22$   
 $y = \frac{22}{x}$ ,  $x = 1$   $y = 22$   
ii)  $x = 100$ ,  $y = \frac{22}{100} = \frac{11}{50}$

- d)  $L = \frac{k}{x}$   
e)  $5 = \frac{k}{6}$ ,  $k = 30$ ,  $L = \frac{30}{x}$ ,  $L = \frac{30}{4} = 7.5$

## NA14iii) Complex Proportion

- a)  $y = kx^2$ ,  $3 = k4^2$ ,  $k = \frac{3}{16}$ ,  $y = \frac{3x^2}{16}$   
 $y = 12$ , so  $12 = \frac{3x^2}{16}$ ,  $x = 8$   
b)  $y = \frac{k}{x^2}$ ,  $8 = \frac{k}{3^2}$ ,  $k = 72$ ,  
 $y = \frac{72}{x^2}$ ,  $18 = \frac{72}{x^2}$ ,  $x = 2$ .

## NA15 Bounds

- a)  $10.5\text{m} \uparrow 9.5\text{m} \downarrow$   
b)  $6.005\text{m} \uparrow 5.995\text{m} \downarrow$   
c)  $1234.5\text{kg} \uparrow 1233.5\text{kg} \downarrow$   
d)  $1234.05\text{kg} \uparrow 1233.95\text{kg} \downarrow$   
e) A = lower + lower, B = lower - upper,  
C = lower  $\times$  lower, D = lower  $\div$  upper  
f) B =  $8.155 - 3.45 = 4.705$   
C =  $8.155 \times 3.55 = 28.95025$

## NA16 Brackets

- a)  $2x + 6$   
b)  $10x - 6$   
c)  $6 - 12x$   
d)  $2x^2 - 10x + 2x - 10$   
 $2x^2 - 8x - 10$   
e)  $x^2 - 5x - 3x + 15$   
 $x^2 - 8x + 15$

## NA17 Factorise i) & ii)

- a)  $8(3x - 1)$   
b)  $2(11x + 4)$   
c)  $(x + 11)(x - 3)$   
d)  $(x - 2)(x - 3)$

## NA18i) Surds

- a)  $2\sqrt{3}$   
b) cannot be simplified  
c)  $\sqrt{35}$   
d)  $5\sqrt{7}$   
e) 12  
f)  $10\sqrt{2}$   
g)  $25 - 12 = 13$

## NA18ii) Surds

- a)  $\frac{\sqrt{5}}{5}$   
b)  $\frac{4\sqrt{7}}{7}$

## NA19 Set-up Eqns

- a)  $x + (x + 10) + (x + 20) + (x + 30) = 360$   
(which would lead to  $x = 75$ )  
b)  $2(x + 3) = 3x + 5$   
(which would lead to  $x = 1$ )

## NA20 Solve Eqns

- a)  $3x - 5 = 8$ ,  $3x = -8 + 5 = -3$ ,  $x = -1$   
b)  $x - 5 = 4$ ,  $x = 9$   
c)  $x = \frac{1}{7}$  or  $1\frac{4}{7}$

## NA21i) & ii) Change Subject

- a)  $y = 3x + 5$

b)  $x = \frac{y + 5}{6}$

c)  $x = \sqrt[3]{y - 2}$

d)  $y^4 = x^3 - 3$

$$y^4 + 3 = x^3$$

$$x = \sqrt[3]{y^4 + 3}$$

## NA21iii) Change Subject

- a)  $2xy - 7x = 5 - y$

$$x(2y - 7) = 5 - y$$

$$x = \frac{5 - y}{2y - 7}$$

- b)  $6x - 2x = 4 + y$

$$x = \frac{4 + y}{4} \text{ or } 1 + \frac{y}{4}$$

- c)  $xy + x = y$

$$x(y + 1) = y$$

$$x = \frac{y}{y + 1}$$

## NA21iii Extra – Complex Expressions

- d)  $y^2 = x - 1$

$$x = y^2 + 1$$

- e)  $y + 1 = \sqrt{x}$

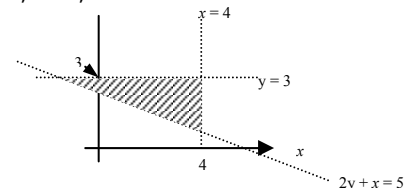
$$x = (y + 1)^2$$

## NA22 Inequalities

- a)  $3x > 27 - 9$ ,  $x > 6$   
b)  $x \leq 30$   
c)  $5x - 3 \geq -28$   
 $5x \geq -28 + 3$   
 $x \geq -5$

## NA23 Regions

- a)  $2y + x = 5$   
 $x = 0$ ,  $y = 2.5$ , crosses y-axis at 2.5  
 $y = 0$ ,  $x = 5$ , crosses x-axis at 5



## NA24 Sequences

- a) 49, 64 b) 64, 128  
c) 100,000 1,000,000  
d) 28, 36 e) 37, 50 . . .  
f) 21, 34 . . .  
g) . . . . .

# Answers – Your Turn!! – Number & Algebra and Shape & Space

## NA25 Trial & Improvement

a) Use  $x^3 - x$

Try 5 → 120

6 → 210

5.9 → 199.479

5.95 → 204.69...

$x = 5.9$  to 1 d.p.

b) try 0.15,  $\frac{1}{0.15} = 6.666...$  too small.

so answer lies between 0.15 & 0.2; 0.2 to 1 d.p.

## NA26 Solving Quadratics by Factorisation

a)  $x = -3$ , or 4

b)  $x+3=0$  when  $x = -3$

$x-4=0$  when  $x = 4$

## NA27i) Completing the Square

a)  $x+9 = \pm 10$ ,  $x = 1$  or  $-19$

b) step 2.  $\frac{B}{2} = \frac{14}{2} = 7$ ,  $(\frac{B}{2})^2 = 49$

step 3.  $(x+7)^2 - 49 + 48 = 0$

step 4.  $(x+7)^2 = 1$

step 5.  $x+7 = \pm 1$

$x = -6$  or  $-8$

## NA27ii) Quadratic Formula

a) -4

b) 4

c) i)  $-4(2)(-6) = 48$

ii)  $\sqrt{16+48} = 8$

d) 2<sup>nd</sup> solution = -3

e)  $a = 16$ ,  $b = 8$ ,  $c = -3$

$x = 0.25$  or  $-0.75$

## NA28 Simultaneous Equations

a) add eq<sup>s</sup>,  $3y = 12$   $y = 4$  and  $x = 3$

b)  $10x + 15y = -30$

(-)  $10x - 8y = 62$

$23y = -92$

$y = -4$  and  $x = 3$

## NA29 Simultaneous Equations with a Quadratic

a) self check

b)  $x = 0$  or 2

$x = 0$  and  $y = 1$  or  $x = 2$  and  $y = 5$

c)  $x - 1 + 1 = x^2 - x$

$0 = x^2 - 2x$

$x = 0$  or 2

$x = 0$  and  $y = -1$  or  $x = 2$  and  $y = 1$

## NA30 n<sup>th</sup> term

a)  $k = 7$ ,  $n = 7n - 4$

b)  $k = -2$ ,  $n = -2n + 102$

## NA31 $y = mx + c$

a) (6,0) (0,2)  $m = \frac{\Delta y}{\Delta x} = \frac{-2}{6} = -\frac{1}{3}$

$c = 2$ ,  $y = -\frac{1}{3}x + 2$  or  $-\frac{x}{3} + 2$

## NA32 Parallel & Perpendicular

a) i) & iii)

b) i) & v) AND iii) & v) AND ii) & iv)

c)  $y = 3x + 4$

## NA33 Graphs

a) C

b) starts to speed up

c) C → D

d) deceleration or slowing down

e) i)

f) QR

g) M → N constant speed (or velocity)

N → O stopped or stationary

## NA34 Plot quads and cubics

a)

$x$	-2	-1	0	1	2
$3x^2$	12	3	0	3	12
$-x$	2	1	0	-1	-2
$-1$	-1	-1	-1	-1	-1
$y = 3x^2 - x - 1$	13	3	-1	1	9

$x$	-2	-1	0	1
$x^2$	4	1	0	1
$-3x$	6	3	0	-3
$-1$	-1	-1	-1	-1
$y = x^2 - 3x - 1$	9	3	-1	-3

b)  $x = 3$   $y = 4(3)^3 + 2(3) - 2 = 112$

$x = -3$   $y = 4(-3)^3 + 2(-3) - 2 = -116$

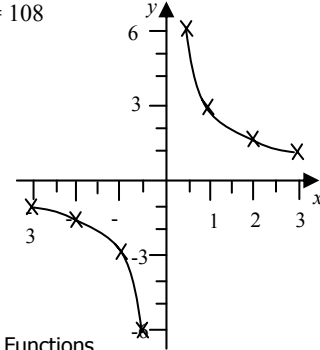
c)  $x = 3$   $y = -4(3)^3 = -108$

$x = -3$   $y = -4(-3)^3 = 108$

## NA35 Reciprocal

$x = 2$   $y = \frac{3}{2}$ ,

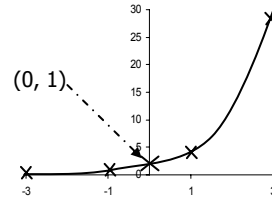
$x = -\frac{1}{2}$   $y = \frac{3}{-\frac{1}{2}} = -6$



## NA36 Exponential & Trig Functions

a)

$x$	-3	-1	0	1	3
$y = 3^x$	$\frac{1}{27}$	$\frac{1}{3}$	1	3	27



b)

$x$	0	90	180	270	360
$y = \cos x$	1	0	-1	0	1

## NA37 Transformations

a)  $f(x)+4$  translation 4 units up

$4f(x)$  stretch  $\times 4$  along y-axis

$f(x+4)$  translation 4 units left

$f(4x)$  stretch  $\times \frac{1}{4}$  along x-axis

b) i) D, ii) C, iii) A, iv) B.

## SHAPE AND SPACE

### SS1 Pythagoras

a) & b) self check

c) i)  $? = \sqrt{9^2 + 12^2} = 15$

ii)  $? = \sqrt{12^2 - 9^2} = \sqrt{63}$  or 7.94 to 3s.f.

### SS2 Trigonometry

a) & b) self check

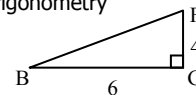
c)  $\cos A = \frac{AB}{AC}$ ,  $\tan A = \frac{BC}{AB}$

d)  $\sin \theta = \frac{4}{8}$ ,  $\theta = 30^\circ$

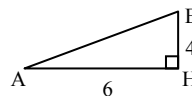
e)  $\cos 38 = \frac{AB}{20}$ ,  $AB = 20 \cos 38 = 15.760.. = 15.8 \text{ cm } 3\text{sf}$

### SS3 3D Pythagoras/Trigonometry

a)



b)  $AE = \sqrt{4^2 + 6^2} = \sqrt{52}$  or 7.21 to 3sf

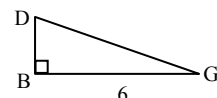


c)

BD can be found from  $\triangle ABD$ ,

$BD = \sqrt{4^2 + 5^2} = \sqrt{41}$  or 6.40 to 3sf

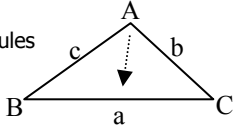
$\tan \theta = \frac{6}{\sqrt{41}}$ ,  $\therefore \theta = 43.1^\circ$  to 3sf



# Answers – Your Turn!! – Shape & Space

## SS4 Sine/Cosine Rules

a)



b)  $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

or not quite as good

$$\cos A = \frac{-b^2 - c^2 + a^2}{-2bc}$$

c) –

d)  $\cos \theta = \frac{2^2 + 5^2 - 6^2}{2 \cdot 2 \cdot 5} = -\frac{7}{20}$       $\theta = 110^\circ$  3sf

## SS5 $\frac{1}{2} ab \sin C$

a)  $\frac{1}{2} \cdot 4.5 \sin 93 = 9.986... = 9.99$  3sf

b) do not have the included angle

## SS6 Bearings

a)  $\angle BCA = \angle BAC = 22.5^\circ$

- b) i)  $225^\circ$   
 ii)  $270^\circ$   
 iii)  $247.5^\circ$

## SS7 Surface Area

a)  $(2 \times 10) \times 4 + (2 \times 2) \times 2 = 88$  units<sup>2</sup>

b) Base =  $16 \times 16 = 256$

$$? = \sqrt{17^2 - 8^2} = 15$$

$$\text{Area } \Delta = \frac{1}{2} \cdot 16 \cdot 15 = 120$$

$$\therefore \text{S.A.} = 256 + 4 \times 120 = 736$$
 units<sup>2</sup>

## SS8 Volume

- a)  $1 \times 2 \times 3 = 6$  cm<sup>3</sup>  
 b)  $\pi \times 3^2 \times 10 = 90\pi$  cm<sup>3</sup> or 283 cm<sup>3</sup> 3sf  
 c) Area  $\Delta = 0.5 \times 6 \times 2 = 6$  cm<sup>2</sup>  
 $V = 6 \times 10 = 60$  cm<sup>3</sup>

## SS9 Circles

- k) no, G is not on circumference  
 n) delete line BOD  
 add line AOC  
 obtuse  $\angle AOC = 2x$  say  
 (acute)  $\angle AOC = 2y$  say  
 $\angle ABC = y$ ,  $\angle ADC = x$ .

## SS10 Areas Sectors/Segments

a) 5.08 cm<sup>2</sup> to 3sf

b) arc length =  $\frac{70}{360} \times 2 \times \pi \times 6 = \frac{7\pi}{3}$

$$AB = 2 \times [6 \sin 35]$$

$$P = 12 \sin 35 + \frac{7\pi}{3} = 14.2$$
 to 3sf

## SS11 Congruence

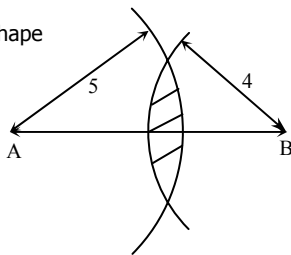
- a) B&C: SAS  
 b) X&Z: ASA or (AAS, SAA)  
 c) XY common  
 $\angle ZXY = \angle WXY (=40^\circ)$   
 $XZ = XW (=6)$   
 $\therefore$  congruent by SAS

## SS12 Constructions

ab) –

## SS13 Loci

- a) Racetrack shape  
 b)



## SS14 Transformations

- a) Reflection in the line  $x = -\frac{1}{2}$

b) i) Translation  $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$

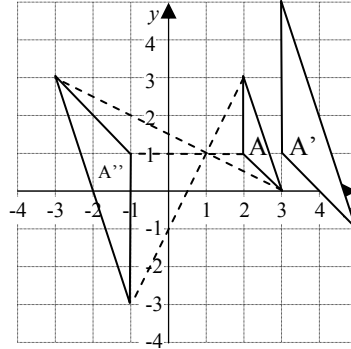
ii) Translation  $\begin{pmatrix} -4 \\ -1 \end{pmatrix}$

- c) i) Rotation  $90^\circ$  anticlockwise (or  $+90^\circ$ ) about  $(-1, 0)$   
 ii) Rotation  $90^\circ$  clockwise (or  $-90^\circ$ ) about  $(-1, 0)$

d) –

## SS15 Enlargement

- a) 5 right (0 up)  
 b)  $\times 2$  centre  $(1, 1)$   
 c)  $\times \frac{1}{2}$  centre  $(1, 1)$   
 d)



- e)  $\times (-\frac{1}{2})$  centre  $(1, 1)$

## SS16 Similar Triangles

- a) IH  
 b) Establish similar  $\Delta$ 's  
 In  $\Delta$ 's ABC & ADE  
 $\angle BAC$  common  
 $\angle ABC = \angle ADE$  F-angles  
 $\angle ACB = \angle AED$  F-angles  
 $\therefore \Delta ABC$  similar to  $\Delta ADE$

$$AC : AE = 5 : 9$$

$$AD = 4 \times \frac{9}{5} = 7.2 \text{ cm}$$

$$DE = 6 \times \frac{9}{5} = 10.8 \text{ cm}$$

## SS17 Area/Volume Similar Shapes

- a)  $20 \times 1.5^3 = 67.5$  g  
 b)  $12 \times 4^3 = 768$  cm<sup>3</sup>

## Extra

c)  $\sqrt[3]{\frac{\text{Big Volume}}{\text{Small Volume}}} = \sqrt[3]{\frac{54}{2}} = 3$

## SS18 Dimensions

- a) Area  
 b) Area  
 c) Area  
 d) Length

## Extra

- e) Area  
 f) Length

## SS19 Speed Density

a)  $D = \frac{M}{V} = \frac{0.414}{90} = 0.0046 \text{ kg/cm}^3$  or  $4.6 \times 10^{-3}$

b)  $D = \frac{M}{V}, 0.001 = \frac{0.05}{V}, V = \frac{0.05}{0.001} = 50 \text{ cm}^3$

## SS20 Converting cm<sup>3</sup> and m<sup>3</sup>

- a)  $30000 \div (100)^3 = 0.03$  m<sup>3</sup>  
 b)  $12 \times (10)^3 = 12,000$  mm<sup>3</sup>

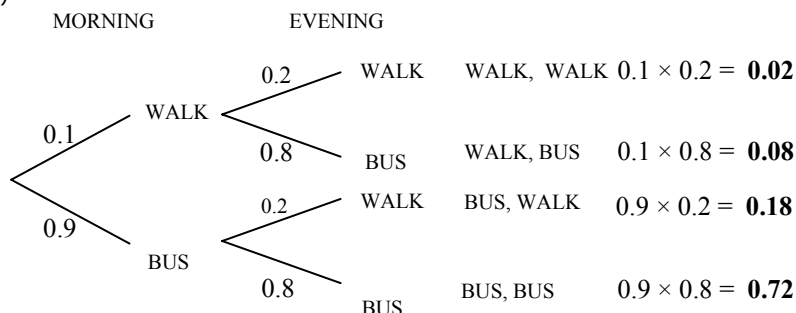
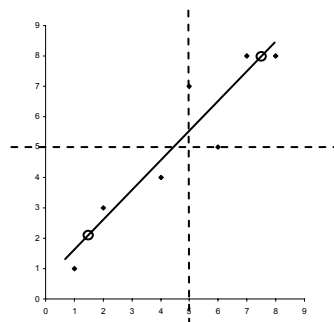
# Answers – Your Turn!! – Handling Data

## HD1 Correlation

- a) 3  
 b) i) (1, 1) (2, 3)  
 ii) (7, 8) (9, 8)  
 c) 3  
 d) positive correlation (moderate)

## HD2 Probability

- a) 75    b) 0.42  
 c)



- c)  $0.02 + 0.08 + 0.18 + 0.72 = 1$   
 d)  $1 - [(\frac{1}{10} \times \frac{2}{15}) + (\frac{2}{10} \times \frac{3}{15}) + (\frac{7}{10} \times \frac{10}{15})] = \frac{12}{75}$

## HD3 Data & Sampling

- b) Introduce bias

c)  $\frac{120}{49000} \times 25000 = 61.224\dots$        $\frac{120}{49000} \times 20000 = 48.979\dots$        $\frac{120}{49000} \times 4000 = 9.7959\dots$

Axbridge → 61

Backwell → 49

Congresbury → 10

- d) each person (in the population) has an equal chance of being selected  
 e) (the population) is divided into strata (or groups)  
 f) Place names of 100 people in a hat and select 10 without looking or number the names of the people from 1-100, and use the random number, RAN calculator button or computer to list 10 numbers from 1-100, select these. e.g. last 2 digits from RAN.

## HD4 Statistical Calculations

a) **Mean** =  $\frac{\text{Sum}}{\text{How Many}}$       **Median** - Middle value      **Mode** - Most Frequent      **Range** = Biggest – Smallest

- b) 1, 1, 1, 1, 1, 2, 2, 2, 2, 5.

Mean =  $\frac{1+1+1+1+1+2+2+2+2+5}{10} = \frac{18}{10} = 1.8$

Median = 1.5 (take mean of 1 and 2)      Mode = 1      Range = 5 - 1 = 4

c)

Weight, w	Frequency	Mid-Point × Frequency
$0 \leq w < 10$	8	$5 \times 8 = 40$
$10 \leq w < 20$	2	$15 \times 2 = 30$
$20 \leq w < 30$	10	$25 \times 10 = 250$
$30 \leq w < 40$	80	$35 \times 80 = 2800$
<i>Total</i>	<i>100</i>	<i>3120</i>

Mean =  $\frac{3120}{100} = 31.2$

Median is the (50 or) 50.5<sup>th</sup> number. This falls in the  $30 \leq w < 40$  group, Modal Group is  $30 \leq w < 40$

- d) Do not have the original numbers, just the range they fall in.

- e) 1, 1, 3, 4, 6, 8, 10, 11, 12, 15, 15.

11 numbers, n = 11.

lower quartile,  $Q_1 = \frac{1}{4}(n+1)^{\text{th}} = 3^{\text{rd}} = 3$

median,  $Q_2 = \frac{1}{2}(n+1)^{\text{th}} = 6^{\text{th}} = 8$

upper quartile,  $Q_3 = \frac{3}{4}(n+1)^{\text{th}} = 9^{\text{th}} = 12$

Inter-Quartile Range,  $Q_3 - Q_1 = 12 - 3 = 9$ .

(Because we have exact numbers we use the (n+1) formula. Only for grouped data and for cumulative frequency graphs where we are looking for an estimate do we use the easier formula where we drop the (n+1) for just n.)

# Answers – Your Turn!! – Handling Data

## HD5 Moving Averages

- a)  $\frac{90+26+146+130}{4} = \frac{392}{4} = 98$   
 $\frac{26+146+130+110}{4} = \frac{412}{4} = 103$   
 $\frac{146+130+110+15}{4} = \frac{401}{4} = 100.25$   
 $\frac{130+110+15+165}{4} = \frac{420}{4} = 105$

## HD6 Cumulative Frequency Curves

a) Extending the table:

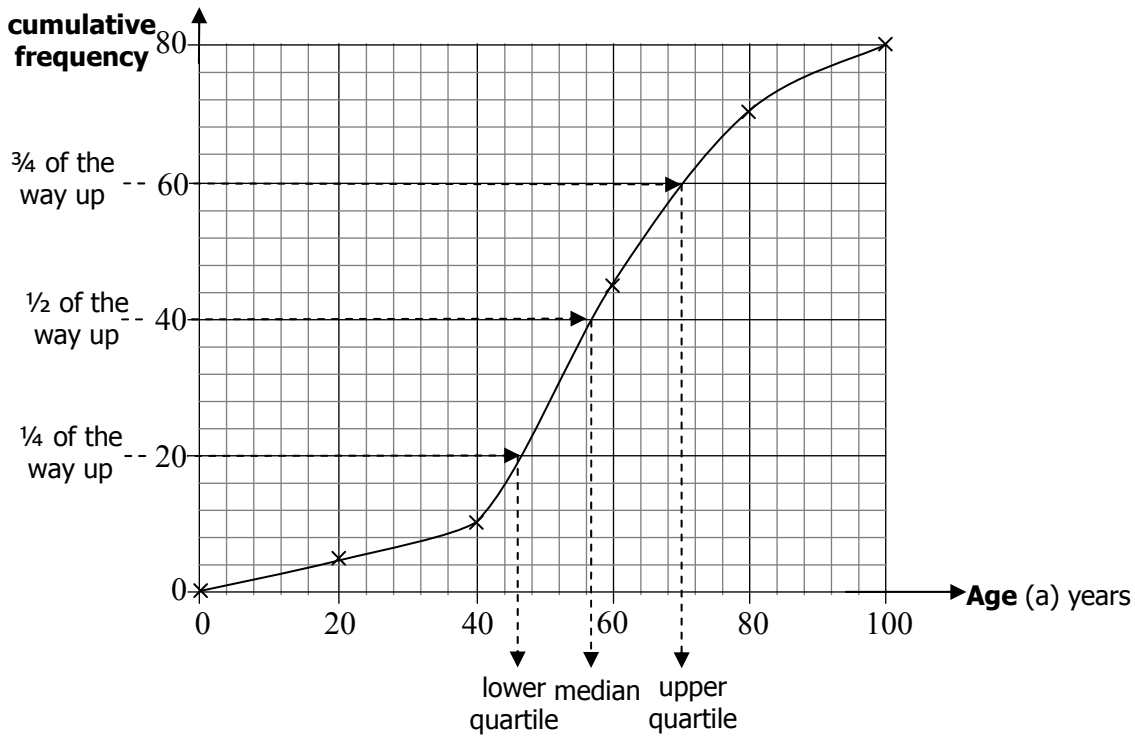
Age (a years)	Frequency	C. F.
$0 \leq a < 20$	5	5
$20 \leq a < 40$	5	10
$40 \leq a < 60$	35	45
$60 \leq a < 80$	25	70
$80 \leq a < 100$	10	80

Or more fully with a new table:

Age, a	C. F.
$< 0$	0
$< 20$	5
$< 40$	10
$< 60$	45
$< 80$	70
$< 100$	80

optional row

- b) - see graph below  
 c) lower quartile  $46 \pm 2$   
 median  $57 \pm 2$   
 upper quartile  $70 \pm 2$



## HD7 Box Plots

- a) lowest value 0.5cm, lower quartile 2.5cm, upper quartile 4.5cm.  
 b) A: Inter-quartile range =  $5.5 - 3.5 = 2$   
 B: Inter-quartile range =  $4.5 - 2.5 = 2$

## HD8 Histograms

a)

Age, a	F	Class Width	F.D. = $\frac{\text{Frequency}}{\text{Class Width}}$
$0 \leq a < 5$	10	5	$10 \div 5 = 2$
$5 \leq a < 10$	20	5	$20 \div 5 = 4$
$10 \leq a < 20$	40	10	$40 \div 10 = 4$
$20 \leq a < 50$	90	30	$90 \div 30 = 3$

- b)  $10 \times 0.8 = 8$ , yes

## HD9 Frequency Density

- a) Frequency = Frequency Density  $\times$  Class Width  
 Frequency =  $0.6 \times 30 = 18$  and  $0.5 \times 40 = 20$

# Answers – Rapid Acid!! – Number & Algebra

## NA1 HCF LCM

- 60
- 4

## NA2 Primes

- (A number with) only two factors
- 2 & 19 are prime
- $2 \times 2 \times 5 \times 5$

## NA3 Negatives

- 32
- 15
- 5
- 2
- 2
- 9

## NA4 Substitution

- $y = 7 - 3(-2) = 7 + 6 = 13$
- $y = 1 - 3(-5)^2 = 1 - 3(25) = 1 - 75 = -74$

## NA5 Index Laws

- $3^{15}$
- $3^{8-7} = 3^1 = 3$
- $3^{8 \times 7} = 3^{56}$
- $x^{3+9} = x^{12}$
- $y^{5-3} = y^2$
- $z^{4 \times 7} = z^{28}$

## NA6 Significant Figures

- 70
- 70.3
- 0.055
- 0.0545

## NA7 SIF

- 13400
- $1.34 \times 10^{-2}$
- $1.522756 \times 10^{12}$

## NA8 Ratio

- $12 : 100 = 3 : 25$
- $21 : 28 = 3 : 4$
- $22 \div 11 = 2$ , so £6, £16
- $100 \div 13 = 7.692307692$   
Jim  $\times 5 = 38.4615\dots$   
= £38.46 to nearest penny

## NA9 Fractions

- $\frac{8}{15}$
- $\frac{2}{3} \times \frac{5}{4} = \frac{1}{3} \times \frac{5}{2} = \frac{5}{6}$
- $1\frac{7}{15}$  or  $(\frac{22}{15})$

## NA10 Fraction Neg. Powers

- a) 2  
b) 8
- a)  $\frac{1}{6}$   
b)  $\frac{1}{25}$   
c)  $\frac{1}{5}$

## NA11 Recurring Decimals

- $\frac{45}{99}$  which simplifies to  $\frac{5}{11}$
- $\frac{402}{999}$  which simplifies to  $\frac{134}{333}$
- $\frac{3}{10}$

## NA12 %'s

- $546 \times 1.175 = £641.55$
- £160
- i)  $400 \times (1.05)^3 = £463.05$   
ii)  $400 \times (1.05)^{30} = £1728.78$  nearest penny

## NA13 Reverse %'s

- £25.74 to nearest penny
- £414.29 to nearest penny

## NA14i) Direct Proportion

- 48
- $\frac{7}{12}$

## NA14ii) Inverse Proportion

- i) 20  
ii)  $\frac{1}{2}$

2.  $yx = 88$  or  $y = \frac{88}{x}$  or equivalent

## NA14iii) Complex Proportion

1.  $y = kx^2$ ,  $9 = k1^2$ ,  $k = 9$ ,  
 $y = 9x^2$ ,  $36 = 9x^2$ ,  $x = 2$

2.  $y = \frac{k}{x^2}$ ,  $9 = \frac{k}{1^2}$ ,  $k = 9$ ,  $y = \frac{9}{x^2}$ ,  $81 = \frac{9}{x^2}$ ,

$x = \frac{1}{3}$

## NA15 Bounds

- a = 6.55kg  $\uparrow$  6.45kg  $\downarrow$   
b = 2.5kg  $\uparrow$  1.5kg  $\downarrow$   
c = 6.45 + 1.5 = 7.95 (kg)  
d = 6.45 - 2.5 = 3.95 (kg)

## NA16 Brackets

- $2 - 6x$
- $2x^2 - 11x + 2x - 11$   
 $2x^2 - 9x - 11$

## NA17 Factorise

- $4(25x - 2)$  note  $2(50x - 4)$  is not fully factorised
- $6(3 - 2x)$
- $(x + 1)(x + 100)$
- $(x + 4)(x - 5)$

## NA18i) Surds

- $1 + \sqrt{3} + \sqrt{3} + 3 = 4 + 2\sqrt{3}$
- $2\sqrt{2}$
- $25\pi \text{ cm}^2$

## NA18ii) Surds

- $\frac{\sqrt{7}}{7}$
- $\frac{4\sqrt{2}}{2} = 2\sqrt{2}$

## NA19 Set-up Eqns

- $3y = y + 5$
- $2(z + 5) = 3z - 4$

## NA20 Solve Eqns

- $3x - 6 + 7 = 1$ ,  $x = \frac{1}{3}$  or  $0.3$
- $x - 2 = 4$ ,  $x = 6$
- $x = \frac{2}{11}$

## NA21i) & ii) Change Subject

- a)  $x = \frac{y - 2}{5}$   
b)  $x = \frac{y + 2}{y}$  or  $1 + \frac{2}{y}$   
c)  $x^3 = 2 - y^2$ ,  $x = \sqrt[3]{2 - y^2}$

## NA21iii) Change Subject

- a)  $x - xy = -1$   
 $x(1 - y) = -1$   
 $x = \frac{-1}{1 - y}$  or  $\frac{1}{y - 1}$   
b)  $xy - x = 1 + 2y$   
 $x(y - 1) = 1 + 2y$

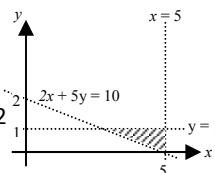
$$x = \frac{1 + 2y}{y - 1}$$

## NA22 Inequalities

- a)  $2x > 6$ ,  $x > 3$   
b)  $x + 3 \geq -27$ ,  $x \geq -30$
- 2, -1, 0, 1

## NA23 Regions

- $2x + 5y = 10$   
 $x = 0$ ,  $y = 2$ , crosses y-axis at 2  
 $y = 0$ ,  $x = 5$ , crosses x-axis at 5



## NA24 Sequences

- $3 \times 81 = 243$
- $7 + 13 + 24 = 44$

## NA25 Trial & Improvement

- Use  $x^3 - 2x - 150$   
Try  $5 \rightarrow -35$  too small  
 $6 \rightarrow 54$  too big  
 $5.5 \rightarrow 5.375$  too big  
 $5.4 \rightarrow -3.336$  too small

**examiners look for half way value i.e. 5.45**  $\rightarrow$   
 $0.978625$  too big

$x = 5.4$  to 1 d.p.

## NA26 Solving Quadratics by Factorisation

- $x = -1$ , or 2
- $(x - 20)(x + 2) = 0$   
 $x = 20$  or  $-2$

## NA27i) Completing the Square

- a)  $(x - 4)^2 - 16$
- b)  $(x - 4)^2 - 16 - 9 = 0$ ,  $x = 9$  or  $-1$

## NA27ii) Quadratic Formula

- a = 4 b = -8 c = -5  
 $x = 2.5$  or  $-0.5$

## NA28 Simultaneous Equations

- $x = -2$   $y = 9$
- $\times 3$   $6x + 9y = 60$   
 $(-)$   $6x + 8y = 44$   
 $y = 16$ ,  $x = -14$

## NA29 Simultaneous Equations with a Quadratic

- $y = 10 - x$   
 $10 - x = x^2 + x + 11$   
 $0 = x^2 + 2x + 1$   
 $0 = (x + 1)(x + 1)$   
 $x = -1$  and  $y = 11$

## NA30 n<sup>th</sup> term

- $-20n + 120$

## NA31 y = mx + c

- gradient,  $m = 3$   
y-intercept,  $c = 4$
- $c = 5$   $m = \frac{\Delta y}{\Delta x} = \frac{4}{-2} = -2$   
 $y = -2x + 5$

## NA32 Parallel & Perpendicular

- i) & iii)
- iv) & v)

## NA33 Graphs

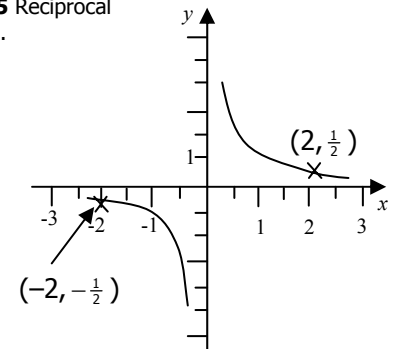
- A  $\rightarrow$  B constant (steady) speed  
B  $\rightarrow$  C deceleration (Speed decreases)
- 8500m
- a) O  $\rightarrow$  P stopped (stationary)  
P  $\rightarrow$  Q constant speed  
b) N  $\rightarrow$  O

## NA34 Plot quads and cubics

x	-2	-1	0	1	2
$3x^3$	-24	-3	0	3	24
$-x^2$	-4	-1	0	-1	-4
<b>+ 1</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>
$y = 3x^3 - x^2 + 1$	<b>-27</b>	<b>-3</b>	1	<b>3</b>	21

## NA35 Reciprocal

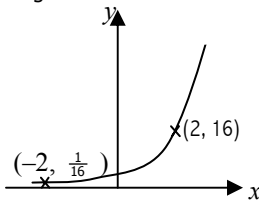
- 



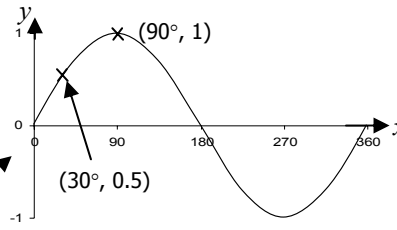
# Answers – Rapid Acid!! – Number & Algebra

## NA36 Exponential & Trig Functions

1.  $y = 4^x$



2.  $y = \sin x^\circ$



## NA37 Transformations

- a) translation 1 unit left  
b)  $y = (x + 1)^2$
- a) stretch  $\times 4$  along y-axis  
b)  $y = 4 \sin x$

# Answers – Rapid Acid!! – Shape & Space

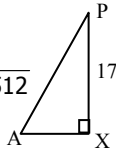
## SS1 Pythagoras

1.  $BC = \sqrt{10^2 - 8^2} = 6$   
 $CD = \sqrt{6^2 + 6^2} = \sqrt{72}$  or 8.49 3sf

## SS2 Trig

1.  $\tan 70 = \frac{AD}{5}$   
 $AD = 5 \tan 70 = 13.73738\dots = 13.7 \text{ cm } 3sf$

2. In  $\triangle ACD$ ,  $\sin C = \frac{AD}{20}$   
 $\sin C = \frac{5 \tan 70}{20} = 0.686869354$   
 $\therefore C = 43.4^\circ$  to 3sf



## SS3 3D Phy/Trig

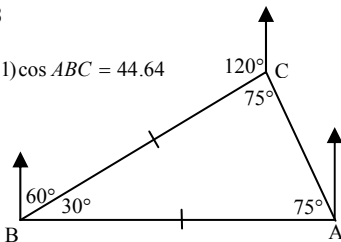
1.  $AC = \sqrt{16^2 + 16^2} = \sqrt{512}$   
 $AX = \frac{1}{2} \sqrt{512}$   
 $AP = \sqrt{17^2 + (\frac{1}{2} \sqrt{512})^2} = \sqrt{417} = 20.4$  (3 sf)

2. Required angle A.

$\tan A = \frac{17}{\frac{1}{2} \sqrt{512}}$ ,  $A = 56.4$  (3sf)

## SS4 3D Phy/Trig

1. a)  $\cos B = \frac{3.1^2 + 3.4^2 - 4.8^2}{2(3.1)(3.4)} = \frac{-1.87}{21.08} = -0.088709677$   
 $\angle CBD = 95.08938013 = 95.1^\circ$  to 3sf  
 b)  $\angle ABC = 180 - 95.08938013 = 84.91061987$   
 $AC^2 = 6.2^2 + 3.1^2 - 2(6.2)(3.1) \cos ABC = 44.64$   
 $AC = 6.68$  3sf



## SS5 $\frac{1}{2} ab \sin C$

1.  $\frac{1}{2} \cdot (4.2)(3.1) \sin 54 = 5.3$  1dp

## SS6 Bearings

1. a)   
 b) Isosceles  $\triangle \therefore \angle ACB = \angle BAC = 75$   
 i)  $345^\circ$  ii)  $360 - 120 - 75 = 165^\circ$

## SS7 Surface Area

1. i)  $2(1 \times 2 + 2 \times 3 + 1 \times 3) = 22 \text{ unit}^2$   
 ii)  $54\pi \text{ unit}^2$  exactly which is  $170 \text{ unit}^2$  to 3sf

## SS8 Volume

- $2 \times 3 \times 4 = 24 \text{ cm}^3$
- $\pi \times r^2 \times h = \pi \times 5^2 \times 6 = 150\pi \text{ cm}^3$  or  $471 \text{ cm}^3$  3sf
- $0.4 \times 0.2 = 0.08 \text{ cm}^3$
- $\frac{1}{3} \times 3^2 \times 5 = 15 \text{ cm}^3$

## SS9 Circles

- $ADC = 180 - 110 = 70$  (iv)
- theorem ii)
- $\angle AOB$  - theorem i)
- i)  $\angle ADB$  - theorem iii)  
ii) acute
- $\angle CAD, \angle CBD$  theorem v)

## SS10 Areas Sectors/Segments

1. Arc length =  $\frac{22}{360} \times 2 \times \pi \times 9$

$P = 18 + \frac{11\pi}{10}$  or 21.5cm to 3sf

2. Area =  $\frac{22}{360} \times \pi \times 9^2 = \frac{99\pi}{20}$  or  $15.6 \text{ cm}^2$  3sf

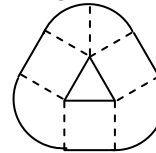
## SS11 Congruence

1. None

## SS12 Constructions

1. Construct  $60^\circ$  - see Equilateral Triangle SS12  
 Bisect this angle - see Angle Bisector SS12

## SS13 Loci



dotted lines are 5cm long

## SS14 Transformations

- a) Reflection  $x = -1/2$   
 b) Translation  $\begin{pmatrix} -3 \\ -5 \end{pmatrix}$   
 c) Rotation about  $(-1, 0)$   $90^\circ$  clockwise or  $(-90^\circ)$

## SS15 Enlargement

- a) scale factor 3 (or  $\times 3$ ) centre (4, 5)  
 b) scale factor  $\frac{1}{3}$  (or  $\times \frac{1}{3}$ ) centre (4, 5)  
 c) scale factor  $-2$  (or  $\times -2$ ) centre (1, 2)  
 d) scale factor  $-1/2$  (or  $\times -1/2$ ) centre (1, 2)

## SS16 Similar Triangles

1. [XY is opposite  $\angle 65^\circ$  this corresponds to side BC]  
 XZ corresponds to AC  
 $\times \frac{15}{6}$  scale factor of enlargement  
 $\therefore XY = 5.6 \times \frac{15}{6} = 14 \text{ cm}$

## SS17 Area/Volume Similar Shapes

1.  $12 \times 3^2 = 108 \text{ cm}^2$

## Extra Optional

2.  $1.2 \times n^2 = 120$ ,  $n = 10$ .  
 $V_B = 2 \times 10^3 = 2000 \text{ cm}^3$

## SS18 Dimensions

- Volume
- Length
- Volume

## SS19 Speed Density

1.  $D = \frac{M}{V} = \frac{16}{400} = 0.04 \text{ kg/cm}^3$

2.  $T = \frac{D}{S} = \frac{110}{20} = 5.5 \text{ s}$

## SS20 Converting $\text{cm}^3$ and $\text{m}^3$

- $0.0025 \times (100)^3 = 2,500 \text{ cm}^3$
- $120 \div (10)^3 = 0.12 \text{ cm}^3$

## SS21 Vectors

1.  $\begin{pmatrix} -3 \\ -4 \end{pmatrix}$

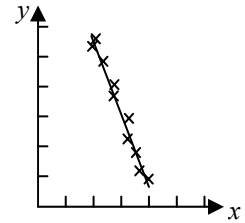
2.  $\vec{CD} = -\vec{b} - \vec{a} + \vec{c} = -\vec{b} - \vec{a} + \vec{b} - \vec{a} = -2\vec{a} = -2\vec{AB}$

So  $\vec{AB}$  is a scalar multiple of  $\vec{CD}$ , so parallel.

# Answers – Rapid Acid!! – Handling Data

## HD1 Correlation

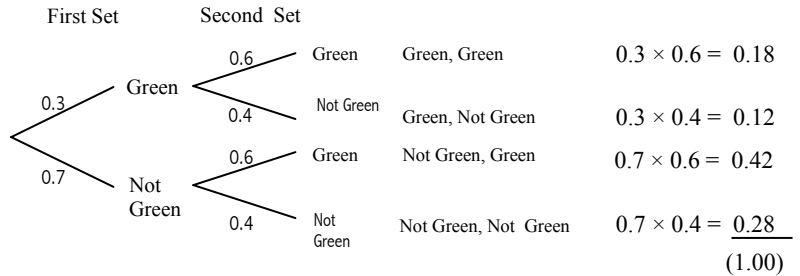
- i) no correlation, a line is NOT appropriate
- ii) (strong) negative correlation



## HD2 Probability

1. By Relative Frequency,  
 $P(\text{first set green}) = \frac{\text{The Number of Successes}}{\text{The Number of Trials}} = \frac{3}{10}$  or 0.3 etc

$P(\text{first set not green}) = 1 - 0.3 = 0.7$



2.  $(\frac{1}{10}) \times (\frac{1}{10}) \times (\frac{1}{10}) \times (\frac{1}{10}) \times (\frac{1}{10})$  or  $(\frac{1}{10})^5$

$= \frac{1}{100,000}$  or 0.00001 or  $1 \times 10^{-5}$

3. a)  $(\frac{2}{10})^3 + (\frac{3}{10})^3 + (\frac{5}{10})^3 = \frac{4}{25}$

b)  $[3 \times (\frac{2}{10})^2 \times \frac{8}{10} + 3 \times (\frac{3}{10})^2 \times \frac{7}{10} + 3 \times (\frac{5}{10})^2 \times \frac{5}{10}] = \frac{33}{50}$

## HD3 Data & Sampling

- $\frac{400}{45000} \times 30000 = 266.666...$        $\frac{400}{45000} \times 10000 = 88.888...$        $\frac{400}{45000} \times 5000 = 44.444...$   
 Dumpy → 267      Eep → 89      Freddie's → 44
- random
- increase size of the sample
- a) data already collected e.g. national census  
 b) leading questions  
 (or by inappropriate selection of people doing the questionnaire including asking too few people)

## HD4 Statistical Calculations

1. a) Mean =  $\frac{30 \times 29 + 50 \times 11 + 70 \times 40 + 90 \times 60}{29 + 11 + 40 + 60} = \frac{9620}{140} = 68\frac{5}{7}$  or 68.7% 3sf

b)  $80 \leq \% < 100$

c) The median is the 70.5<sup>th</sup> number, this falls in the  $60 \leq \% < 80$  group.

(For grouped data taking the 70<sup>th</sup> number is also fine, giving the same answer.)

2. 1, 1, 4, 5, 6, 8, 10, 11, 12, 30. (10 numbers)

lower quartile is the  $\frac{1}{4}(n+1)^{th} = 2.75 \rightarrow 3^{rd} = 4$ .

upper quartile is the  $\frac{3}{4}(n+1)^{th} = 8.25 \rightarrow 8^{th} = 11$ .

IQR =  $11 - 4 = 7$ .

## HD5 Moving Averages

1. a)  $\dots, \frac{32}{7}, \frac{33}{7}, \frac{36}{7}, \frac{38}{7}, \frac{43}{7}, \frac{47}{7}$ .

b) Thursday (of the first week).

## HD6 Cumulative Frequency Diagrams

1. optional cf table

Score (%)	cf
< 20	0
< 40	4
< 60	12
< 80	18
< 100	20

lower q =  $44 \pm 2$

median =  $55 \pm 2$

upper q =  $69 \pm 2$

## HD7 Box Plots

- a) i) 1cm  
 ii) 13cm  
 b) i) 5 cm, 9cm  
 ii)  $9 - 5 = 4$ cm  
 c) 6cm

## HD8 Histograms

Score (%)	F	Class Width	F.D. = $\frac{\text{Frequency}}{\text{Class Width}}$
$0 \leq \% < 30$	12	30	$12 \div 30 = 0.4$
$30 \leq \% < 50$	8	20	$8 \div 20 = 0.4$
$50 \leq \% < 60$	15	10	$15 \div 10 = 1.5$
$60 \leq \% < 100$	20	40	$20 \div 40 = 0.5$

## HD9 Frequency Density

- frequency density of  $0 \leq a < 10$ ,  $40 \div 10 = 4$ ;  
 so  $10 \leq a < 40$  would have a frequency density of 8  
 and a frequency of  $8 \times 30 = 240$ .  
 [by ratio of area  $\times 6$  ( $= 40 \times 6 = 240$ )]

